Shaft CenterLINES

Misalignment and Shaft Crack-Related Phase Relationships for 1X and 2X Vibration Components of Rotor Responses

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1. Introduction

The most significant components of the rotating machine lateral vibrational responses, as measured by XY proximity transducers, and eventually filtered, are 1X synchronous vibration and the vibration with twice rotative speed frequency (2X). The major part of the 1X synchronous vibration is due to rotor unbalance. If the rotor is lightly unbalanced, not heavily preloaded, and the rotor is stable (no fluid-induced, or rub-induced, or other vibrations), the 1X vibration is the only dominant component in the rotor response.

The twice rotative speed (2X) component appears in the rotor vibrational spectrum as a result of two main causes:

i) Nonlinearity

ii) Asymmetry in the rotating system.

Nonlinearity, mainly the nonlinearity of the rotor/bearing stiffness characteristics, becomes important when the rotor lateral deflections are high. These deflections can be caused by large unbalance, radial preload force, or both.

Asymmetry in the rotating system or, as it is often called, anisotropy, mainly refers to the different rotor stiffnesses in two perpendicular lateral directions. For instance, in the "X" direction the nonrotating shaft is weaker, and deflects more under the given radial load force, and in the perpendicular to X, the "Y" direction the shaft is stronger, and deflects less under the same, but perpendicularly to the former applied force. This rotor stiffness asymmetry might result from geometric causes, such as non-circular cross-section shaft, unevenly restrained rotor by press-fitted parts on it or, finally, by a shaft lateral crack.

This article discusses two main cases when 1X and 2X vibrational components are dominant in the rotor response. The differences in the rotor vibrational response components 1X and 2X in each case help to diagnose the machine malfunction.

2. Phase Rules

The 1X phase α_1 as measured by digital vector filters, is by definition as follows (Figure 1):

"The 1X phase is the distance from the Keyphasor® pulse to the first positive peak of 1X time-base wave form."

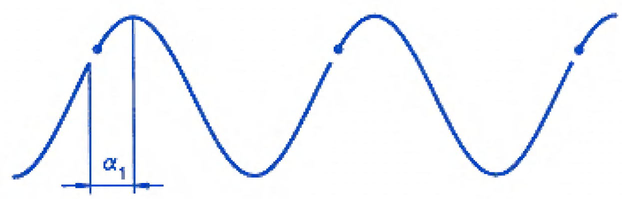


Figure 1
1X response timebase wave form

The formula for the 1X wave has the following form:

$$x = A_1\cos(\omega t + \alpha_1) \quad (1)$$

where α_1 is the phase lag , i.e., it inexplicitly carries the sign minus.

The 2X vibrations component phase α_2, as measured by digital vector filters, is by definition as follows (Figure 2):

"The 2X phase is the distance from the Keyphasor pulse to the first positive peak of 2X time-base wave from."

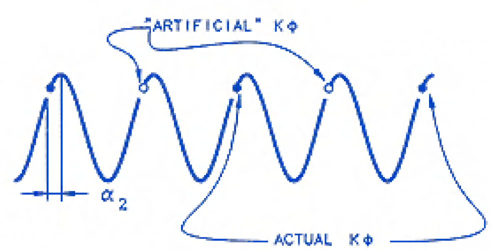


Figure 2
2X response timebase wave form

NOTE that on the 2X wave there are actual Keyphasor dots and "artificial" dots in between, so the phase, α_2, can vary from 0 to 360 degrees of the 2X wave period (which is from 0 to 180 degrees of the 1X wave period). The artificial Keyphasor dots are considered of equal value to the actual ones.

The formula for the 2X wave is as follows:

$$x = A_2\cos(2\omega t + \alpha_2) \quad (2)$$

where again, α_2 is the phase lag , so it inexplicitly carries the minus sign.

3. Orbits Versus Lissajous Figures

The **Lissajous** figures are particular cases of orbits. The classical Lissajous figures result from two perpendicular (XY orientation) ***harmonic*** motions with ***two*** different frequencies. Note that rotor orbital motion results from two perpendicular motions which ***both*** are ***sums*** of harmonic motions, and usually carry the same set of frequencies. Thus, if "x" contains 1X and 2X components, then most probably "y" will have 1X and 2X components, though with different amplitudes and phases. The magnitude of the component amplitudes and the phase relationships determine the final shape of the orbit.

A few examples of pure Lissajous figures are given below:

Suppose the rotor horizontal motion is represented by a 1X component only, thus

$$x = A \cos \omega t \qquad (3)$$

with $\alpha_1 = 0°$ for simplicity.

Assume that in the rotor vertical motion the dominant is a 2X component (neglect the others), thus

$$y = B \cos (2\omega t + \beta_2) \qquad (4)$$

where β_2 is the 2X component phase.

Depending on the value of β_2 the resulting orbits will have different shapes. Assume, for example, that $\alpha_2 = 45°$ (lag). In Figure 3 it is shown how the orbit is built up from the "x" and "y" wave forms. The resulting orbit is known as a "butterfly" or "rabbit ears". In the orbit construction the time steps are labeled 1, 2, 3, ..., etc. Thus the direction of orbital motion can easily be determined.

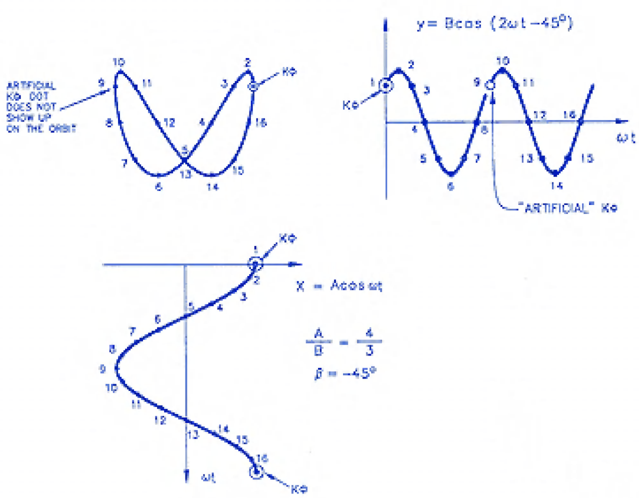
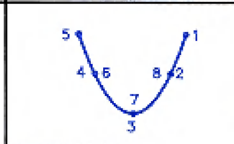
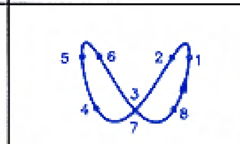
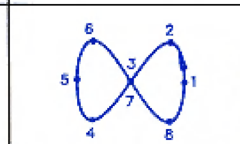
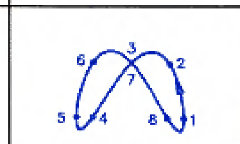
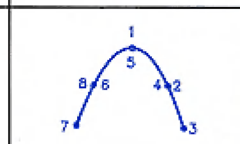
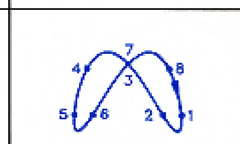
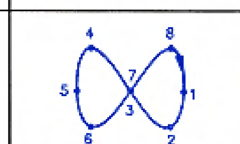
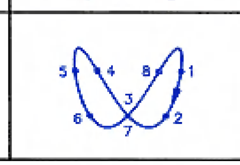


Figure 3
Building the orbit from horizontal x and vertical y wave forms

The other orbits resulting from different angles β_2 are given in Table 1. The numbers on the orbits indicate time steps.

Table 1

Case	β_2 (lag) [Degrees]	Vertical Response y	Orbit (with horizontal response x = Acosωt. A = 4B/3)
1	0	B cos 2 ωt	
2	45	B cos (2ωt−45)	
3	90	B cos (2ωt−90)	
4	135	B cos (2ωt−135)	
5	180	B cos (2ωt−180)	
6	225	B cos (2ωt−225)	
7	270	B cos (2ωt−270)	
8	315	B cos (2ωt−315)	

Note that in all cases the orbits have outside loops (except the extreme cases for $\beta_2 = 0°$ or $\beta_2 = 180°$). This happens because there is no 1X component in the "y" wave.

Now try to introduce a 1X component in the "y" wave with 90 degrees phase difference from the "x" wave, which is most often the case, resulting in circular or symmetric elliptical 1X orbits. First, assume the 1X component having a small amplitude, then, for the following cases, gradually increasing amplitudes. The corresponding orbits are shown in Figure 4. ▶

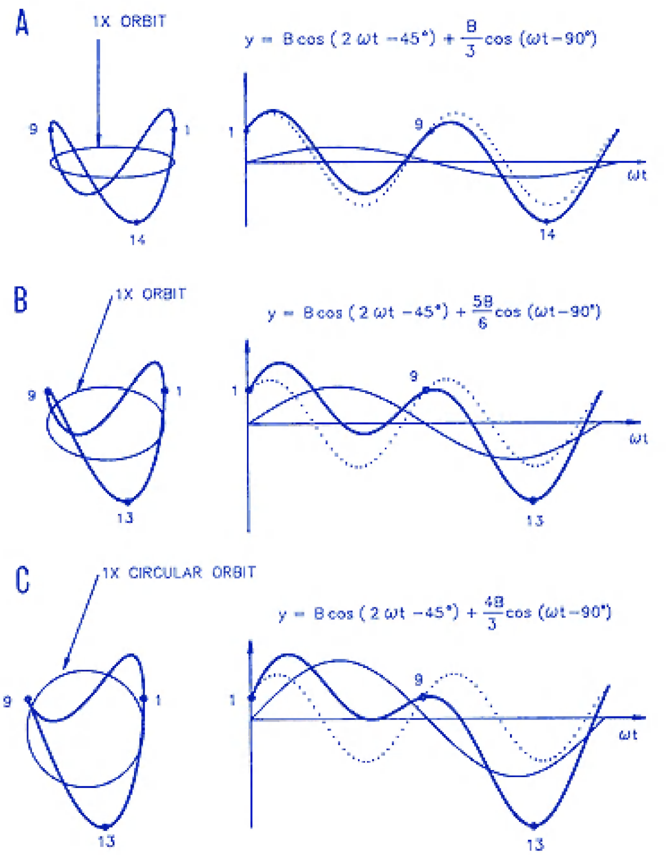


Figure 4
Orbits and vertical time-base wave (the horizontal time-base wave for all cases is the same: $x = (4B/3)\cos \omega t$)

In all cases illustrated in Figure 4 the same as in Figure 3 "x" component was maintained, namely

$x = A \cos \omega t = (4B/3)\cos \omega t$

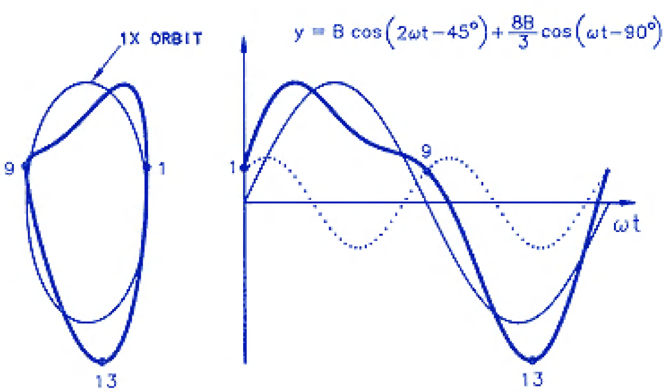


Figure 5
Orbit and vertical time-base wave with a significant 1X component ($x = (4B/3)\cos\omega t$).

The pure 1X orbits are shown overlayed (dotted line) on the resulting orbits. As it can be seen, a small 1X component in the "y" wave does not significantly modify the orbit, in comparison to the case illustrated in Figure 3. With an increase of 1X amplitude in the "y" response, the external loop on the orbit decreases, and eventually vanishes. The existence of the 2X component in the orbit is only marked by a point at one side, and elongation at the other side.

In the last case presented in Figure 5, the existence of a 2X component is hardly noticed.

The magnitude of 1X in the vertical component depends, for instance, on the radial preload in the vertical direction: higher preload, results in lower amplitude of the 1X "y" component, as the shaft gets bent and/or is pushed to the side of a bearing.

4. Sources of 1X and 2X components

The 1X component in the rotor vibration response spectrum is mainly due to unbalance. The existence of 2X component in the rotor spectrum is mainly due to two factors:

1) Shaft misalignment (including gear mesh and belt drives) and resulting radial preload, or radial preload generated by fluid flow.
2) Shaft asymmetry (such as with cracked shafts) together with radial preload (from misalignment, fluid flow, gravity or other origins).

These two cases are physically different. In the first case, 1X and 2X components are closely related, as the only significant factors are the unbalance plus radial preload, and nonlinearity of the system stiffness. Due to the preload, the shaft is forced to rotate in a bent configuration. This involves the effect of stiffness nonlinearity in the response. The shaft rotates in the range of deflections where the stiffness of supports also behave nonlinearily (fluid film stiffness in bearings, and seals in particular). The region of deflection is determined by the specific direction of the radial preload force, thus the rotor responses in vertical and horizontal directions differ.

In the second case the preload also plays a role. However, the second component is not the system stiffness nonlinearity, but stiffness anisotropy (asymmetry), and only of those elements which are involved in rotative motion (mainly the rotor).

In both cases, most of the 1X component is produced by unbalance. In the first case, it becomes modified by the radial preload thereby resulting in rotor displacement to the side. In these circumstances the stiffness nonlinearity causes not only changes in the 1X response magnitudes, but is also responsible for the generation of higher harmonics, as secondary vibrational components. Thus, in the first case, the main vibration excitation factor is solely unbalance. The preload and nonlinearity modify the unbalance response.

In the second case, the situation is different. There is still unbalance, and the unbalance-related 1X response. The preload does not have to be high, i.e., does not push the shaft to nonlinear ranges of deflections. The moderate preload together with rotating anisotropy of shaft stiffness causes another excitation factor, ***independently*** from unbalance. Since the stiffness varies twice per one rotation of the shaft, this excitation has a 2X frequency. The mechanism is purely linear (assuming stiffness does not change).

In the first case, the 2X component was secondary to the 1X component, thus its amplitude is most often much smaller than that of 1X. In the second case the 2X component is independent from 1X, thus its amplitude might appear much larger than that of 1X (especially at 2X resonance).

So far the discussion concerned only the 1X and 2X amplitudes. The phase relationships are certainly of equal importance. The phase relationships may assist in an appropriate identification of the vibrational signal, and the correct diagnosis of the machine problem. These phase relationships for both problems are discussed below.

5. Case 1 – "Misalignment"

When the radial load is ***vertical up***, the unbalance/nonlinearity response of the horizontal rotor can roughly be presented as follows, [1]:

Horizontal:

$$x = A_x \cos(\omega t + \alpha_x + 90°) \tag{5}$$

Vertical:

$$y = A_{oy} + A_1 \cos(\omega t + \alpha_1) + A_2 \cos(2\omega t + \alpha_2) \tag{6}$$

where "A_{oy}" is the static displacement of the shaft centerline due to the vertical load. Assuming a weak cross-coupling between "x" and "y", the horizontal response "x" remains pure 1X due to the original unbalance. Amplitude A_1 of the vertical response becomes smaller than A_x, as the rotor is displaced to the top and restricted in its motion. The second harmonic component appears in the vertical response due to nonlinearity. The phases of the vertical response versus rotative speed are illustrated in Figure 6, where "δ" is the angular location of unbalance ("heavy spot"). Note that since the components of the vertical motion are expressed in terms of cosines, the heavy spot appears with -90 degree angle: this should be compared with the horizontal response phase α_x in Figure 7.

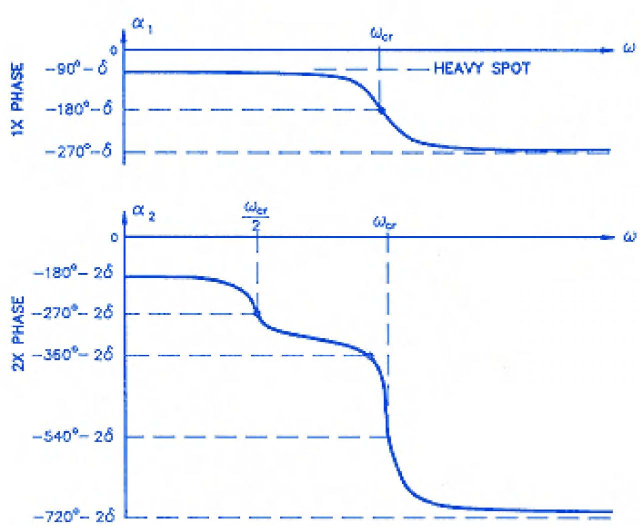


Figure 6

1X and 2X phases of the rotor vertical response (6) versus rotative speed. δ is the angular location of the unbalance ("heavy spot") [1].

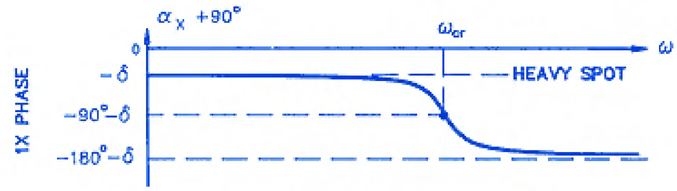


Figure 7

1X phase of the rotor horizontal response (5) versus rotative speed. δ is the angular position of the unbalance ("heavy spot").

The phases α_1 and α_x practically differ very little. In Figures 6 and 7, ω_{cr} denotes first balance resonance speed. Note that the 2X component of the vertical response has two resonances, one at $\omega_{cr}/2$ and the second at ω_{cr}! The phase varies 540 degrees.

6. *Case 2 – "Cracked Shaft"*

The response of an anisotropic rotor to vertical load is roughly as follows, [2]:

$$x = A_{ox} + A_1 \cos(\omega t + \alpha_1 + 90°) + A_2 \cos(2\omega t + \alpha_2 + 90°) \tag{7}$$

$$y = A_{oy} + A_1 \cos(\omega t + \alpha_1) + A_2 \cos(2\omega t + \alpha_2) \tag{8}$$

where "A_{ox}" and "A_{oy}" are shaft static displacements due to the radial load. In comparison to the " misalignment" case, here both "x" and "y" have similar components: the shaft statically moves in both "x" and "y" directions. Both 1X and 2X components are circular (the same amplitudes and 90 degree phase differences). The 1X and 2X phases of responses (7) and (8) are illustrated in Figure 8.

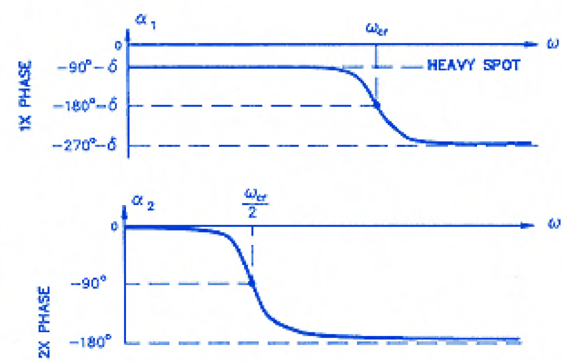


Figure 8

1X and 2X phase of the rotor horizontal and vertical response (7) and (8) versus rotative speed [2].

Note that in both cases "Misalignment" and "Cracked Shaft" the phases of the 1X components are practically the same. The significant difference concerns the 2X phase component here, as compared to the "Misalignment" case.

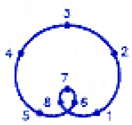
7. Comparison of the "Misalignment" and "Cracked Shaft"-Related Orbits

Now the question arises as to what kind of orbits should be expected in both "Misalignment" and "Cracked Shaft" cases. A significant difference is that in the first case the x,y responses are not matching, while in the second case they are, and produce circular orbits. In the first case, the external loops in the orbits are quite probable. The internal loops are characteristic for ►

circular motions in the same direction which is the "Cracked Shaft" case. Since unbalance-related 1X vibration is "forward" the 2X component should also be "forward", to produce an internal loop orbit. An example is given below.

Following the "Cracked Shaft" case, assume that in equations (7) and (8) the amplitudes and phases of the 1X and 2X components are as follows: $A_1 = (5/6)A_2$ and $\alpha_1 = -90°$, $\alpha_2 = -180°$.

The resulting orbit is shown in Figure 9.

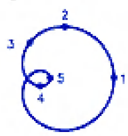


$$x = A_2\left[\frac{5}{6}\cos(\omega t) + \cos(2\omega t - 90°)\right]$$

$$y = A_2\left[\frac{5}{6}\cos(\omega t - 90°) + \cos(2\omega t - 180°)\right]$$

Figure 9
"Cracked Shaft" orbit with 2X phase equal to −180 degrees.

When the 2X phase angle changes to $\alpha_2 = -90°$, for instance due to a change in the load direction, the shape of the orbit remains the same, but the loop rotates 90 degrees as illustrated in Figure 10.

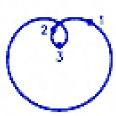


$$x = A_2\left[\frac{5}{6}\cos(\omega t) + \cos(2\omega t)\right]$$

$$y = A_2\left[\frac{5}{6}\cos(\omega t - 90°) + \cos(2\omega t - 90°)\right]$$

Figure 10
"Cracked Shaft" orbit with 2X phase equal to −90 degrees.

Finally, when the 2X phase angle changes to $\alpha_2 = 0°$ the loop rotates another 90 degrees as shown in Figure 11.



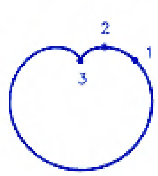
$$x = A_2\left[\frac{5}{6}\cos(\omega t) + \cos(2\omega t + 90°)\right]$$

$$y = A_2\left[\frac{5}{6}\cos(\omega t - 90°) + \cos(2\omega t)\right]$$

Figure 11
"Cracked Shaft" orbit with 2X phase equal to 0 degrees.

Since both 1X and 2X orbits are circular and rotate in the same direction, the phases α_1 and α_2 affect only the position of the internal loop. The amplitude magnitudes, or rather the magnitude of their ratio A_1/A_2, predicts whether there will be a full internal loop, or only an indentation on the 1X orbit. For $A_1 > 2A_2$, the internal loop does not exist. An example illustrated in Figure 12 explains the point. This example should be compared with the previous one, illustrated in Figure 11.

$A_1 = A_2$, $\alpha_1 = -90°$, $\alpha_2 = 0°$



$$x = A_2\left[2\cos(\omega t) + \cos(2\omega t + 90°)\right]$$

$$y = A_2\left[2\cos(\omega t - 90°) + \cos(2\omega t)\right]$$

Figure 12
"Cracked Shaft" orbit for higher ratio of 1X to 2X amplitudes; $\alpha_2 = 0°$.

In Section 3, pure Lissajous orbits for 1X synchronous vibration in horizontal direction and 2X vibration in vertical direction were discussed. Note that in the corresponding equations (3) and (4) the 1X phase was supposed zero, thus β_2 in Equation (4) represented the pure phase difference between 2X and 1X components.

The equations (5) and (6) for the "Misalignment" case can be transformed to the same format. Introduce a new variable:

$$\omega_1 t = \omega t + \alpha_x + 90° \tag{13}$$

Substitute (13) into Equations (5) and (6):

$$x = A_x \cos \omega_1 t \tag{14}$$

$$y = A_{oy} + A_1\cos(\omega_1 t + \alpha_1 - \alpha_x - 90°) + A_2\cos(2\omega_1 t + \alpha_2 - 2\alpha_x - 180°) \tag{15}$$

The phase difference between 2X and 1X components is, therefore

$$\beta_2 = \alpha_2 - 2\alpha_x - 180°$$

For the "Misalignment" case the 2X and 1X phase difference β_2 is illustrated in Figure 13.

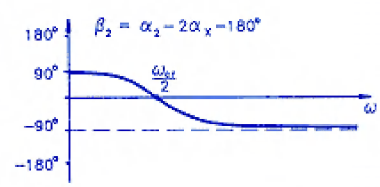


Figure 13
2X and 1X phase difference for the "Misalignment" case versus rotative speed.

The angle β_2 varies, from +90 to −90 degrees, passing through zero when $\omega = \omega_{cr}/2$ (half speed resonance).

For the "Misalignment" case the orbits can be similar to those presented in Table 1. Since $\beta_2 = 270°$ lag (Case 7 in Table 1) corresponds to $\beta_2 = +90°$, at low rotative speed the orbit similar to Case 7 can be expected. At $\omega = \omega_{cr}/2$ (half-speed resonance), the orbit of Case 1 (Table 1) occurs. For high rotative speeds, the orbits evolve through Case 2 to Case 3 (Table 1).

Note that in practical cases of "Misalignment", the 1X component in the vertical wave does exist. Therefore, the orbits will be modified, in comparison to the pure Lissajous figures from Table 1. This was explained in Section 3.

8. Closing Remarks

This article discusses the differences in rotor lateral vibration synchronous (1X) and twice rotative speed frequency (2X) responses for two cases: (i) high radial preload on the rotor, resulting in high eccentricity and involvement of stiffness nonlinearity, and (ii) lower radial preload on the rotor, together with rotor asymmetry. The information helps in correct diagnosis of the rotating machine malfunction.

References

1. Muszynska, A., Misalignment Model, BRDRC, 1989.
2. Muszynska, A., Shaft Crack Detection, Seventh Machinery Diagnostics Seminar, National Research Council, Edmonton, Alberta, Canada, 1982.